Results

Equations (6) and (7) have been evaluated for $x_{c.g.}/l = 0.5$ using the curve for $n + \frac{1}{2}\theta_c(\partial n/\partial\theta_c)$ shown in Fig. 2 and for three bluntness ratios, $\xi = 0.1$, 0.2, and 0.3. The calculated derivatives are plotted in Fig. 3 vs C, and the Newtonian values from Eqs. (8) and (9) are included. The damping derivatives C_{m_q} as defined in this note‡ depend mainly on C and are practically independent of ξ for the chosen c.g. position. For other c.g. locations the variations of C_{m_q} with ξ is greater. The static derivatives C_{m_α} depend on C and also on ξ. Some recent experimental results from small amplitude forced oscillation tests at zero trim angle of attack, Mach 10, $\theta_c = 10°$, $\xi = 0.15$, and 0.3[6] are also shown in Fig. 3. The trend of decreasing derivatives found experimentally for cones with values of $C < 0.7$ is predicted by the theory. Quantitatively the experimental derivatives are lower than the theoretical values. Note that the obvious agreement of one or the other measured derivative with the Newtonian prediction is purely accidental. The evident Reynolds number effect on the damping derivatives indicates that there is an additional viscous effect that is not included in the present theory. Tests are presently being conducted at the Aerospace Research Laboratory to verify experimentally the assumptions and results of this theory.

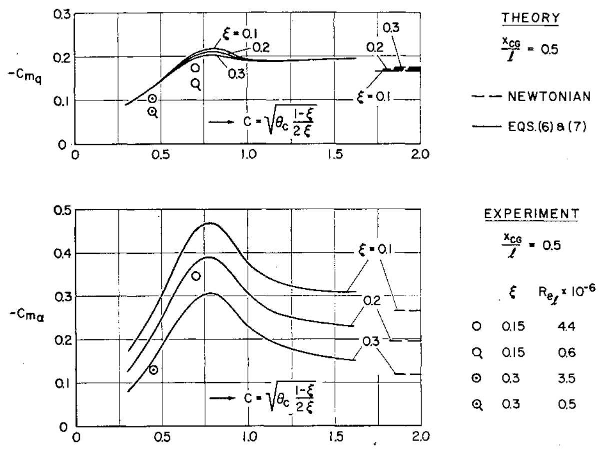

Fig. 3 Stability derivatives vs C, evaluated from Eqs. (6–9) for $x_{c.g.}/l = 0.5$. Comparison with some experiments.[6]

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‡ Some authors use the cone base diameter as the reference length for defining C_{m_q} and C_{m_α}, and some use the actual cone length. The correlation shown herein does not exist when C_{m_q} and C_{m_α} are defined by using the base diameter.

Thermodynamics of Turbine and Piston Rankine Space Power Cycles

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Nomenclature

P = pressure, atm
T_B = boiler temperature, °F
T_C = condenser temperatures, °F
V_F = final specific volume, ft³/lb
V_0 = initial specific volume at boiler conditions, ft³/lb
η = thermal cycle efficiency

The usual alkali metal space power cycle is the Rankine, with continuous flow turbines. However, noncontinuous flow machines, e.g., the conventional steam piston engine, are possible. Several types of noncontinuous flow MHD generators are being studied[1] in which either a liquid metal or MHD lubricated solid piston is driven in reciprocating motion through a magnetic field by metal vapor. These generators are mechanically and electrically attractive, but compactness requires high operating pressures and low expansion ratios, with some irreversible throttling to the condenser. A study was made comparing thermal efficiency and weight for turbines and pistons with varying expansion ratios. Potassium was chosen as a typical alkali metal working fluid. Its thermodynamic properties are the best known, but still quite incomplete. The T-S diagram was constructed with the following data and assumptions:

1) The saturated liquid T-S curve was drawn using C_s data from work at Battelle[2] with extrapolation above 2100°F.

2) Critical constants were taken from Grosse.[3]

3) Entropy of vaporization was assumed equal to experimental values for mercury at corresponding reduced temperatures, following Grosse.[3]

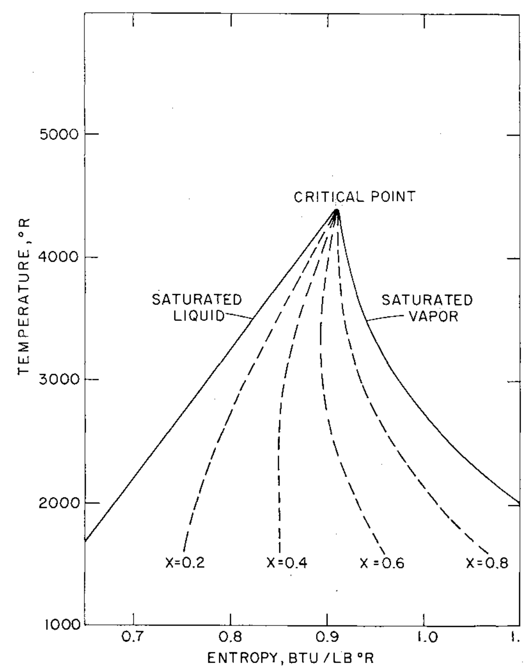

Fig. 1 *T-S* diagram for potassium.

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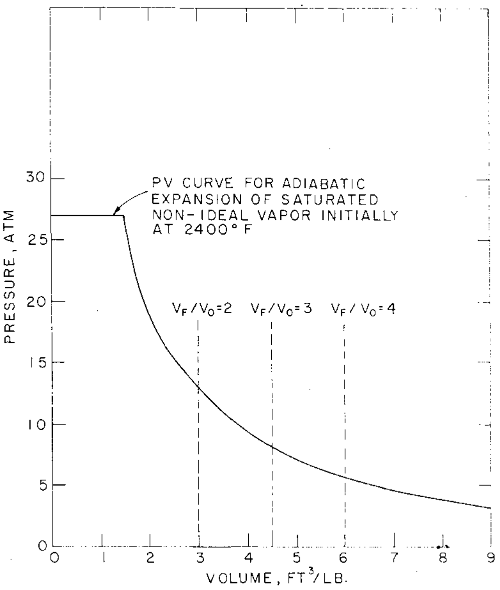

Fig. 2 *PV* relation for potassium vapor expansion. *T* boiler = 2400°F.

4) Vapor pressures were taken from work at Battelle[4] with extrapolation above 2100°F. Compressibility factors and vapor specific volumes were derived from the Clapeyron equation.

The resultant *T-S* diagram (Fig. 1) substantially agrees with that constructed at Battelle[5] up to the latter's limit of 2100°F and extends the range to the critical point.

The thermal efficiency for a turbine was calculated assuming a reversible-adiabatic expansion of the nonideal vapor to the condenser pressure. Piston thermal expansion efficiencies were calculated according to

$$\eta = \frac{(P_s - P_c)V_0 + \int_{V_0}^{V_F} (P - P_c)dV}{\int_{T_C}^{T_B} C_s dT + \lambda_{T_B}}$$

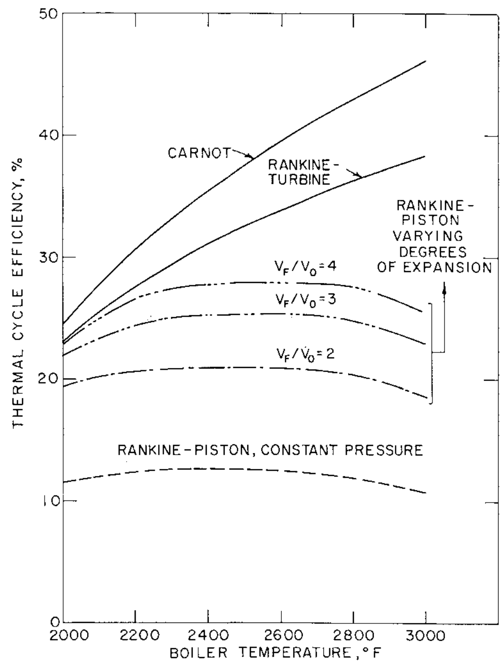

Fig. 3 Thermal cycle efficiency for potassium vapor expansion. *T* condenser = 1400°F.

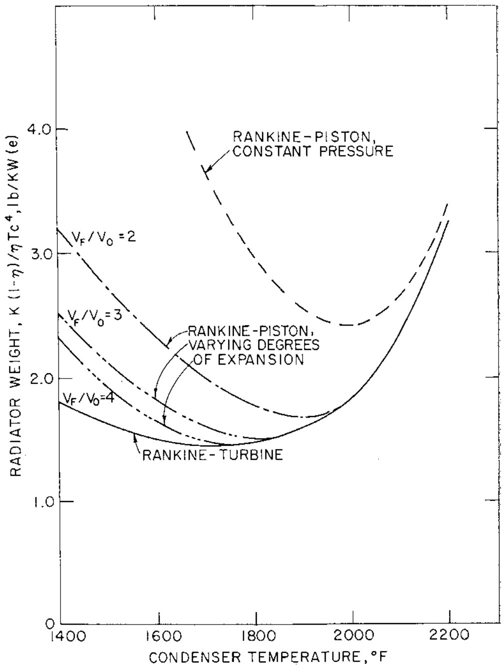

Fig. 4 Radiator weight for potassium vapor expansion. *T* boiler = 2400°F.

which assumes a reversible-adiabatic expansion to a volume V_F and subsequent Joule-Thompson throttling of the exhaust gas to the condenser (Fig. 2).

Resultant efficiencies are shown in Fig. 3 for variable boiler temperatures and a fixed condenser temperature of 1400°F. Note that a constant pressure piston expansion has about 50% of the turbine efficiency, and 2/1 volume expansion of about 80%. This indicates that the major part of the *PV* work is developed for low expansion ratios. Higher expansion ratios have efficiencies even closer to the turbine. The piston efficiency curves all have a maximum with temperature. This is a result of the increasing discrepancy between condenser

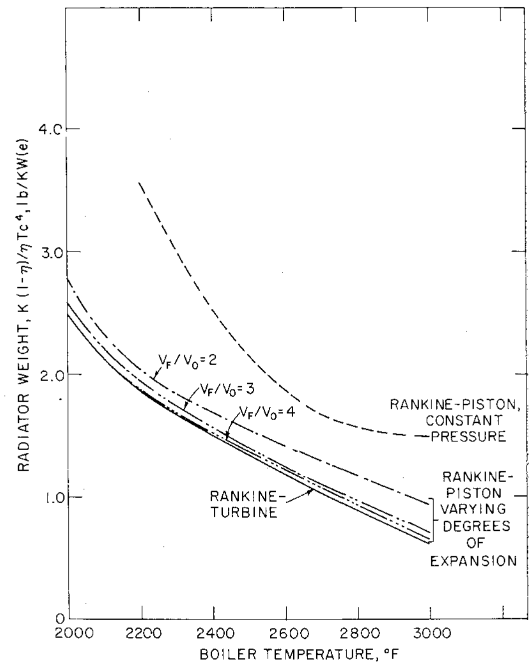

Fig. 5 Locus of minimum radiator weights for potassium vapor expansion.

pressure and exhaust pressure at the end of the stroke as temperature increases.

For space power cycles, a more important parameter than thermal efficiency is system weight. Since the major part of the system is radiator weight, turbine and piston were compared with regard to a radiator weight factor, $(1 - \eta)/\eta T_c^4$, which is normalized to a given design. It represents the area, and hence weight, needed to reject the waste heat from the cycle. Figure 4 shows the effect of varying condenser temperature on this factor for a fixed boiler temperature of 2400°F. There is a minimum in the curve because $\eta \to 0$ as $T_c \to T_B$ and $T_c^4 \to 0$ as $T_c \to 0$. The minimum weight for the turbine expansion corresponds to a piston expansion of $V_F/V_0 = 5/1$. However, a piston expansion of V_F/V_0 of 2/1 adds only 15% more weight, because the lower efficiency is offset by a higher condenser temperature.

Figure 5 shows the locus of all the minimum radiator weights for different expansions and boiler temperatures. Again note that a piston expansion of 2/1 closely approaches a turbine expansion. A 2/1 expansion MHD generator would weigh about 0.65 lb/kw(e) as compared to 0.5 lb/kw(e) for the constant pressure generator.

Thus, piston devices of low expansion ratios, e.g., 2/1, can approach, within a few percent, thermal efficiencies and system weights of turbines for Rankine alkali metal space power cycles.

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A Simple Approach to the Multicomponent Laminar Boundary Layer

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Owing to the difficulties usually encountered in numerical solutions of the boundary-layer equations, there exists a necessity for approximate methods by which one may obtain over-all boundary-layer characteristics. One of the more prominent approaches to this problem is to consider that the flow is locally similar. The validity of the approach relies on the condition that the properties of the fluid at the external and body-surface boundaries are slowly varying functions of the streamwise coordinate.

The method presented herein is based on this local similarity premise as applied to the multicomponent laminar boundary layer with either frozen or equilibrium chemistry prevailing. Accordingly, it is possible to transform the governing conservation equations to the form

$$(Cf'')' + ff'' = (2\xi/u_e)(du_e/d\xi)[f'^2 - (\rho_e/\rho)] \tag{1}$$

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$$(Cg')' + fg' = 0 \tag{2}$$

$$(Cz_i')' + fz_i = 0 \tag{3}$$

by utilizing the Levy-Lees transformation

$$\xi = \int_0^s \rho_* \mu_* u_e r^{2j} ds \qquad \eta = \frac{\rho_e u_e r^j}{(2\xi)^{1/2}} \int_0^y \frac{\rho}{\rho_e} dy$$

and considering unit values of the Prandtl and Lewis numbers and Υ_{ie} equal to a constant.

In order to obtain a solution for the system [Eqs. (1–3)], it is necessary to specify the coupling existing between the density and enthalpy. If, in accordance with the approach of Ref. 1, a new parameter $\phi(\eta)$ is introduced, defined by

$$\phi(\eta) = (g - \lambda)/(1 - \lambda) \tag{4}$$

and an empirical relation of the form

$$\rho_e/\rho = (1 - \kappa)f'^2 + \kappa\phi \tag{5}$$

is adopted, there results for Eqs. (1–3)

$$f''' + ff'' = \beta(f'^2 - \phi) \tag{6}$$

$$\phi'' + f\phi' = 0 \tag{7}$$

$$z_i'' + fz_i' = 0 \tag{8}$$

subject to conditions

$$f(0) = f'(0) = 0 \qquad \phi(0) = \phi_w \qquad z_i(0) = z_{iw} \tag{9}$$

$$f', \phi, z_i \to 1 \qquad f'', \phi', z_i' \to 0 \quad \text{as } \eta \to \infty \tag{10}$$

when it is assumed that $C = \rho\mu/\rho_*\mu_* = 1$ and $\beta = (2\kappa\xi/u_e) \times (du_e/d\xi)$ is a constant. It is evident from Eqs. (7) and (8) that a Crocco-type relation exists between $\phi(\eta)$ and $z_i(\eta)$, i.e.,

$$z_i = [1/(1 - \phi_w)][(z_{iw} - \phi_w) + (1 - z_{iw})\phi] \tag{11}$$

and that the solutions to Eqs. (6) and (7) subject to boundary conditions (9) and (10) are those given by Cohen and Reshotko.[2] The constants λ and κ of Eqs. (4) and (5) are to be determined by insuring the satisfaction of Eq. (5) at as many points in the boundary layer as possible.

Fitting the Density Ratio

Since Eq. (5) is identically satisfied at the outer edge of the layer, λ and κ can be evaluated by matching the density and its derivative at the body surface. Hence, there results[1]

$$\lambda = (h_w/H_e) - (c_{pw}/\alpha_w H_e) \tag{12}$$

$$\kappa = (\rho_e/\rho_w)[1 + (\alpha_w/c_{pw})(H_e - h_w)] \tag{13}$$

where

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad \alpha = \rho\left(\frac{\partial(1/\rho)}{\partial T}\right)_p = \frac{1}{T}\left[1 - \frac{T}{W}\left(\frac{\partial W}{\partial T}\right)_p\right] \tag{14}$$

Consistent with the local similarity assumption, λ, κ, and β, which are in general functions of streamwise location, are considered locally to be specific constants varying from each locally similar section to the next. The history of the downstream flow is retained only in the ξ dependence of the similarity parameter η. Furthermore, heat-transfer correlations have indicated that the reference conditions should be based on the local reference enthalpy value, which is compatible with the local similarity approach.

Specification of the Physical Pressure Gradient

Using the momentum equation at the outer edge and the transformation conditions, it is possible to express β according to

$$\beta = -2\kappa \frac{dp}{ds} \frac{\int_0^s \rho_* \mu_* u_e r^{2j} ds}{\rho_e u_e^3 \rho_* \mu_* r^{2j}} \tag{15}$$